

ADDENDUM TO ‘A NEW RESULT ON THE PROBLEM OF BURATTI, HORAK AND ROSA’

ANITA PASOTTI AND MARCO ANTONIO PELLEGRINI

We report in the following tables a linear realization of some lists  $L = \{1^a, 2^b, 3^c, 5^d\}$  used by the authors to prove the results of [1], omitted there for sake of brevity. We order the lists according to the Lemmas/Propositions/Theorems in which they are used. In the second column of the following tables, we write P if the corresponding linear realization is perfect.

LEMMA 3.3

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(0, 2v + 4, 1, 2)$	$[0, \dots, 2v, 2v + 5, 2v + 7, 2v + 2, 2v + 4, 2v + 6, 2v + 3, \dots, 1]$
$(0, 2v + 5, 1, 2)$	$[0, \dots, 2v + 4, 2v + 7, 2v + 5, 2v + 3, 2v + 8, 2v + 6, 2v + 1, \dots, 1]$
$(0, 2v + 3, 2, 1)$	$[0, \dots, 2v, 2v + 5, 2v + 2, 2v + 4, 2v + 6, 2v + 3, \dots, 1]$
$(0, 2v + 4, 2, 1)$	$[0, \dots, 2v + 2, 2v + 7, 2v + 5, 2v + 3, 2v + 6, 2v + 4, 2v + 1, \dots, 1]$
$(0, 3, 3t + 4, 1)$	$[0, 2, \dots, 3t + 2, 3t + 7, \dots, 1, 3, \dots, 3t + 6, 3t + 8, 3t + 5]$
$(0, 3, 3t + 5, 1)$	$[0, 2, \dots, 3t + 8, 3t + 3, 3t + 6, 3t + 9, 3t + 7, \dots, 1, 3, \dots, 3t]$
$(0, 3, 3t + 6, 1)$	$[0, 2, \dots, 3t + 8, 3t + 10, 3t + 7, 3t + 4, 3t + 9, \dots, 3, 1, \dots, 3t + 1]$
$(0, 3, 3t + 2, 2)$	$[0, 5, \dots, 3t + 5, 3t + 7, \dots, 7, 2, 4, 1, 3, \dots, 3t + 6]$
$(0, 3, 3t + 3, 2)$	$[0, 5, \dots, 3t + 8, 3t + 6, \dots, 3, 1, 4, 2, 7, \dots, 3t + 7]$
$(0, 3, 3t + 7, 2)$	$[0, 3, 1, 4, 6, \dots, 3t + 6, 3t + 11, \dots, 2, 7, \dots, 3t + 10, 3t + 12, 3t + 9]$
$(0, 4, 3t + 1, 1)$	$[0, 2, \dots, 3t + 5, 3t + 3, \dots, 3, 1, \dots, 3t + 1, 3t + 6, 3t + 4]$
$(0, 4, 3t + 2, 1)$	$[0, 2, \dots, 3t + 5, 3t + 7, 3t + 4, 3t + 6, 3t + 1, \dots, 1, 3, \dots, 3t + 3]$
$(0, 4, 3t + 3, 1)$	$[0, 2, \dots, 3t + 2, 3t + 7, 3t + 5, 3t + 8, 3t + 6, \dots, 3, 1, \dots, 3t + 4]$
$(0, 4, 3t + 2, 2)$	P $[0, 5, \dots, 3t + 5, 3t + 7, \dots, 7, 2, 4, 1, 3, \dots, 3t + 6, 3t + 8]$
$(0, 4, 3t + 3, 2)$	P $[0, 5, \dots, 3t + 8, 3t + 6, \dots, 3, 1, 4, 2, 7, \dots, 3t + 7, 3t + 9]$
$(0, 4, 3t + 4, 2)$	$[0, 2, \dots, 3t + 5, 3t + 10, 3t + 7, 3t + 9, 3t + 6, 3t + 4, \dots, 1, 3, \dots, 3t + 3, 3t + 8]$

## LEMMA 3.6

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(0, 2v + 5, 0, 3)$	$[0, \dots, 2v, 2v + 5, 2v + 7, 2v + 2, 2v + 4, 2v + 6, 2v + 8, 2v + 3, \dots, 1]$ ,
$(0, 2v + 6, 0, 3)$	$[0, \dots, 2v + 4, 2v + 9, 2v + 7, 2v + 5, 2v + 3, 2v + 8, 2v + 6, 2v + 1, \dots, 1]$
$(0, 5, 0, 5k + 1)$	$[0, \dots, 5k, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 6, \dots, 1, 3, \dots, 5k + 3, 5k + 5]$
$(0, 7, 0, 5k + 6)$	$[0, \dots, 5k + 5, 5k + 3, \dots, 3, 1, \dots, 5k + 6, 5k + 8, 5k + 13, 5k + 11, 5k + 9, \dots, 4, 2, \dots, 5k + 12, 5k + 10]$
$(0, 7, 0, 5k + 3)$	$[0, 2, 4, \dots, 5k + 4, 5k + 6, \dots, 1, 3, \dots, 5k + 8, 5k + 10, \dots, 5, 7, \dots, 5k + 7, 5k + 9]$
$(0, 7, 0, 5k + 7)$	$[0, \dots, 5k + 10, 5k + 12, 5k + 14, 5k + 9, 5k + 7, \dots, 2, 4, \dots, 5k + 4, 5k + 6, \dots, 1, 3, \dots, 5k + 13, 5k + 11]$

## PROPOSITION 3.7

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(0, 1, 3t + 6, 4)$	$[0, 3, 8, \dots, 3t + 11, 3t + 9, \dots, 6, 1, 4, 7, 2, 5, 10, \dots, 3t + 10]$
$(0, 1, 3t + 8, 4)$	$[0, 3, 6, 1, 4, 9, \dots, 3t + 12, 3t + 7, \dots, 7, 2, \dots, 3t + 11, 3t + 13, 3t + 10]$
$(0, 1, 3t + 4, 5)$	$[0, 5, 2, 7, \dots, 3t + 10, 3t + 8, \dots, 8, 3, 6, 1, 4, 9, \dots, 3t + 9]$
$(0, 1, 3t + 6, 5)$	$[0, 3, 8, \dots, 3t + 11, 3t + 6, \dots, 6, 1, 4, 7, 2, 5, 10, \dots, 3t + 10, 3t + 12, 3t + 9]$
$(0, 1, 4, 5k + 2)$	$[0, 3, \dots, 5k + 3, 5k + 5, \dots, 5, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 6]$
$(0, 1, 5, 5k + 7)$	$[0, 3, \dots, 5k + 13, 5k + 10, \dots, 5, 2, \dots, 5k + 12, 5k + 9, 5k + 11, \dots, 1, 4, \dots, 5k + 4]$
$(0, 1, 6, 5k + 7)$	$[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 14, 5k + 12, \dots, 2, 5, \dots, 5k + 10, 5k + 13, 5k + 8, 5k + 11]$
$(0, 1, 7, 5k + 7)$	$[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 9, 5k + 12, \dots, 2, 5, \dots, 5k + 15, 5k + 13, 5k + 8, 5k + 11, 5k + 14]$
$(0, 1, 4, 5k + 4)$	$[0, 3, \dots, 5k + 8, 5k + 5, \dots, 5, 2, \dots, 5k + 7, 5k + 9, \dots, 4, 1, \dots, 5k + 6]$
$(0, 1, 5, 5k + 4)$	$[0, 3, \dots, 5k + 8, 5k + 10, \dots, 5, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 9]$
$(0, 1, 6, 5k + 4)$	$[0, 3, \dots, 5k + 8, 5k + 11, 5k + 9, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 5, \dots, 5k + 10]$
$(0, 1, 7, 5k + 4)$	$[0, 3, \dots, 5k + 8, 5k + 11, 5k + 6, 5k + 9, 5k + 12, 5k + 7, 5k + 10, \dots, 5, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 1]$
$(0, 1, 8, 5k + 4)$	$[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 9, 5k + 12, \dots, 2, 5, \dots, 5k + 5, 5k + 8, 5k + 11, 5k + 13, 5k + 10]$
$(0, 1, 4, 5k + 1)$	$[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 2, \dots, 2, 5, \dots, 5k + 5]$
$(0, 1, 5, 5k + 1)$	$[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 7, 5k + 5, \dots, 5, 2, \dots, 5k + 2]$
$(0, 1, 6, 5k + 1)$	$[0, 3, \dots, 5k + 3, 5k + 6, 5k + 8, 5k + 5, \dots, 5, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 1]$
$(0, 1, 7, 5k + 6)$	$[0, 3, \dots, 5k + 13, 5k + 10, \dots, 5, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 11, 5k + 14, 5k + 12, 5k + 9]$
$(0, 1, 5, 5k + 3)$	$[0, 3, \dots, 5k + 8, 5k + 5, \dots, 5, 2, \dots, 5k + 7, 5k + 9, 5k + 6, \dots, 1, 4, \dots, 5k + 4]$
$(0, 1, 6, 5k + 8)$	$[0, 3, \dots, 5k + 13, 5k + 15, \dots, 5, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 11, 5k + 14, 5k + 9, 5k + 12]$
$(0, 1, 7, 5k + 3)$	$[0, 3, \dots, 5k + 8, 5k + 11, 5k + 9, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 7, 5k + 10, \dots, 5, 2, \dots, 5k + 2]$
$(0, 1, 6, 5k + 5)$	$[0, 3, \dots, 5k + 8, 5k + 11, \dots, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 5, \dots, 5k + 10, 5k + 12, 5k + 9]$
$(0, 1, 7, 5k + 5)$	$[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 9, 5k + 12, \dots, 2, 5, \dots, 5k + 10, 5k + 13, 5k + 11, 5k + 8]$

## PROPOSITION 3.8

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(0, 2, 3t + 4, 3)$	$[0, 2, \dots, 3t + 8, 3t + 3, \dots, 3, 1, \dots, 3t + 1, 3t + 6, 3t + 9, 3t + 4, 3t + 7]$
$(0, 2, 3t + 5, 3)$	P $[0, 2, \dots, 3t + 8, 3t + 3, \dots, 3, 1, \dots, 3t + 1, 3t + 6, 3t + 9, 3t + 4, 3t + 7, 3t + 10]$
$(0, 2, 3t + 6, 3)$	$[0, 2, \dots, 3t + 5, 3t + 10, \dots, 1, 3, \dots, 3t + 3, 3t + 8, 3t + 11, 3t + 6, 3t + 9]$
$(0, 2, 3t + 3, 4)$	$[0, 2, \dots, 3t + 2, 3t + 7, 3t + 4, 3t + 9, 3t + 6, 3t + 1, \dots, 1, 3, \dots, 3t + 3, 3t + 8, 3t + 5]$
$(0, 2, 3t + 4, 4)$	$[0, 3, 1, 6, \dots, 3t + 9, 3t + 4, \dots, 4, 2, \dots, 3t + 2, 3t + 7, 3t + 10, 3t + 5, 3t + 8]$
$(0, 2, 3t + 5, 4)$	P $[0, 3, 1, 6, \dots, 3t + 9, 3t + 4, \dots, 4, 2, \dots, 3t + 2, 3t + 7, 3t + 10, 3t + 5, 3t + 8, 3t + 11]$
$(0, 2, 2, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 3]$
$(0, 2, 3, 5k + 1)$	P $[0, \dots, 5k + 5, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 3, 5k + 6]$
$(0, 2, 4, 5k + 1)$	P $[0, 2, \dots, 5k + 2, 5k + 5, \dots, 5, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 7]$
$(0, 2, 5, 5k + 1)$	$[0, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 5, \dots, 5k + 5, 5k + 8, 5k + 6]$
$(0, 2, 6, 5k + 1)$	P $[0, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 5, \dots, 5k + 5, 5k + 8, 5k + 6, 5k + 9]$
$(0, 2, 2, 5k)$	$[0, \dots, 5k, 5k + 3, \dots, 3, 1, \dots, 5k + 1, 5k + 4, \dots, 4, 2, \dots, 5k + 2]$
$(0, 2, 3, 5k)$	P $[0, \dots, 5k, 5k + 3, \dots, 3, 1, \dots, 5k + 1, 5k + 4, \dots, 4, 2, \dots, 5k + 2, 5k + 5]$
$(0, 2, 4, 5k + 5)$	$[0, 2, \dots, 5k + 7, 5k + 10, \dots, 5, 3, \dots, 5k + 8, 5k + 11, 5k + 6, 5k + 9, \dots, 4, 1, \dots, 5k + 1]$
$(0, 2, 5, 5k + 5)$	P $[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 9, 5k + 11, 5k + 8, 5k + 10, \dots, 5, 2, \dots, 5k + 12]$
$(0, 2, 4, 5k + 2)$	P $[0, 3, \dots, 5k + 3, 5k + 5, \dots, 5, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 8]$
$(0, 2, 5, 5k + 2)$	$[0, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 4, \dots, 5k + 9, 5k + 6, 5k + 8, 5k + 5, \dots, 5, 2, \dots, 5k + 7]$
$(0, 2, 6, 5k + 2)$	P $[0, 3, \dots, 5k + 8, 5k + 5, \dots, 5, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 9, 5k + 7, 5k + 10]$
$(0, 2, 7, 5k + 2)$	$[0, 3, \dots, 5k + 3, 5k + 6, 5k + 9, 5k + 11, 5k + 8, 5k + 10, \dots, 5, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 1]$
$(0, 2, 3, 5k + 4)$	$[0, 3, \dots, 5k + 8, 5k + 6, \dots, 1, 4, \dots, 5k + 9, 5k + 7, \dots, 2, 5, \dots, 5k + 5]$
$(0, 2, 4, 5k + 4)$	$[0, 2, 5, \dots, 5k + 10, 5k + 7, \dots, 7, 4, \dots, 5k + 9, 5k + 6, \dots, 1, 3, \dots, 5k + 8]$
$(0, 2, 5, 5k + 4)$	P $[0, 2, 5, \dots, 5k + 10, 5k + 7, \dots, 7, 4, \dots, 5k + 9, 5k + 6, \dots, 1, 3, \dots, 5k + 8, 5k + 11]$
$(0, 2, 4, 5k + 3)$	$[0, 2, 5, \dots, 5k + 5, 5k + 8, \dots, 3, 1, \dots, 5k + 6, 5k + 9, \dots, 4, 7, \dots, 5k + 7]$
$(0, 2, 5, 5k + 3)$	P $[0, 2, 5, \dots, 5k + 5, 5k + 8, \dots, 3, 1, \dots, 5k + 6, 5k + 9, \dots, 4, 7, \dots, 5k + 7, 5k + 10]$
$(0, 2, 6, 5k + 3)$	$[0, 2, 5, \dots, 5k + 10, 5k + 7, \dots, 7, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 8, 5k + 11, 5k + 6, 5k + 9]$

THEOREM 2.3

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(0, 3, 1, 5k)$	$[0, \dots, 5k, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 3]$
$(0, 3, 1, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 3, \dots, 3, 1, \dots, 5k + 1, 5k + 4, \dots, 4, 2, \dots, 5k + 2]$
$(0, 3, 1, 5k + 2)$	$[0, \dots, 5k + 5, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 6, \dots, 1, 3, \dots, 5k + 3]$
$(0, 3, 1, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 8, \dots, 3, 1, \dots, 5k + 6, 5k + 4, \dots, 4, 2, \dots, 5k + 7]$
$(0, 3, 2, 5k)$	$[0, 2, \dots, 5k + 2, 5k + 5, \dots, 5, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 4, \dots, 5k + 4]$
$(0, 3, 2, 5k + 1)$	$[0, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 3, \dots, 3, 5, \dots, 5k + 5]$
$(0, 3, 2, 5k + 2)$	$[0, 2, \dots, 5k + 7, 5k + 5, \dots, 5, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 4]$
$(0, 3, 2, 5k + 3)$	$[0, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 8, \dots, 3, 5, \dots, 5k + 5]$
$(0, 3, 2, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 8, \dots, 3, 1, \dots, 5k + 6, 5k + 9, 5k + 7, \dots, 2, 4, \dots, 5k + 4]$
$(0, 3, 3, 5k)$	$[0, 2, \dots, 5k + 2, 5k + 5, \dots, 5, 3, \dots, 5k + 3, 5k + 6, 5k + 4, \dots, 4, 1, \dots, 5k + 1]$
$(0, 3, 5, 5k + 2)$	$[0, 2, \dots, 5k + 2, 5k + 5, \dots, 5, 3, \dots, 5k + 8, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 7]$
$(0, 3, 3, 5k + 3)$	$[0, 2, \dots, 5k + 7, 5k + 9, \dots, 4, 1, \dots, 5k + 6, 5k + 3, \dots, 3, 5, \dots, 5k + 5, 5k + 8]$
$(0, 3, 3, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 8, 5k + 10, 5k + 7, \dots, 2, 4, \dots, 5k + 9, 5k + 6, \dots, 1, 3, \dots, 5k + 3]$
$(0, 3, 3, 5k + 6)$	$[0, 2, \dots, 5k + 12, 5k + 9, \dots, 4, 1, \dots, 5k + 11, 5k + 8, 5k + 10, \dots, 5, 3, \dots, 5k + 3]$
$(0, 3, 4, 5k)$	$[0, 2, 5, \dots, 5k + 5, 5k + 7, \dots, 7, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 3, 5k + 6]$
$(0, 3, 4, 5k + 1)$	$[0, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 4, \dots, 5k + 4, 5k + 6, 5k + 8, 5k + 5, \dots, 5, 2, \dots, 5k + 7]$
$(0, 3, 4, 5k + 2)$	$[0, 2, 5, \dots, 5k + 5, 5k + 8, \dots, 3, 1, \dots, 5k + 6, 5k + 9, 5k + 7, \dots, 7, 4, \dots, 5k + 4]$
$(0, 3, 4, 5k + 3)$	$[0, 3, \dots, 5k + 8, 5k + 10, \dots, 5, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 9, 5k + 7]$
$(0, 3, 4, 5k + 4)$	$[0, 2, \dots, 5k + 7, 5k + 10, \dots, 5, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 9, 5k + 11, 5k + 8]$
$(0, 3, 5, 5k + 2)$	$[0, \dots, 5k + 5, 5k + 8, 5k + 10, 5k + 7, \dots, 2, 4, 1, 3, \dots, 5k + 3, 5k + 6, \dots, 6, 9, \dots, 5k + 9]$
$(0, 4, 1, 5k + 3)$	$[0, 2, \dots, 5k + 7, 5k + 5, \dots, 5, 3, \dots, 5k + 8, 5k + 6, \dots, 1, 4, \dots, 5k + 4]$
$(0, 4, 2, 5k + 3)$	$[0, 2, 4, \dots, 5k + 9, 5k + 6, \dots, 1, 3, \dots, 5k + 8, 5k + 5, \dots, 5, 7, \dots, 5k + 7]$
$(0, 4, 3, 5k + 3)$ P	$[0, 2, 4, \dots, 5k + 9, 5k + 6, \dots, 1, 3, \dots, 5k + 8, 5k + 5, \dots, 5, 7, \dots, 5k + 7, 5k + 10]$
$(0, 4, 4, 5k + 3)$	$[0, 3, \dots, 5k + 8, 5k + 10, 5k + 7, 5k + 9, 5k + 11, \dots, 1, 4, \dots, 5k + 4, 5k + 2, \dots, 2, 5, \dots, 5k + 5]$
$(0, 4, 5, 5k + 3)$	$[0, 3, 5, 2, \dots, 5k + 7, 5k + 9, 5k + 12, 5k + 10, \dots, 10, 8, \dots, 5k + 8, 5k + 11, \dots, 1, 4, \dots, 5k + 4]$
$(0, 5, 1, 5k + 3)$	$[0, 2, 4, \dots, 5k + 9, 5k + 7, \dots, 7, 5, \dots, 5k + 5, 5k + 8, \dots, 3, 1, \dots, 5k + 6]$
$(0, 5, 2, 5k + 3)$	$[0, \dots, 5k + 10, 5k + 8, 5k + 6, 5k + 9, 5k + 7, \dots, 2, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 3]$
$(0, 5, 3, 5k + 3)$	$[0, 2, \dots, 5k + 7, 5k + 10, 5k + 8, 5k + 5, \dots, 5, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 4, \dots, 5k + 9, 5k + 11, 5k + 6]$
$(0, 5, 4, 5k + 3)$	$[0, 2, \dots, 5k + 12, 5k + 9, 5k + 11, 5k + 8, 5k + 10, \dots, 5, 3, \dots, 5k + 3, 5k + 6, 5k + 4, \dots, 4, 1, \dots, 5k + 1]$
$(0, 6, 1, 5k + 3)$	$[0, 2, \dots, 5k + 7, 5k + 9, \dots, 4, 1, \dots, 5k + 1, 5k + 3, \dots, 3, 5, \dots, 5k + 10, 5k + 8, 5k + 6]$
$(0, 4, 1, 5k + 2)$	$[0, \dots, 5k + 5, 5k + 7, \dots, 2, 4, \dots, 5k + 4, 5k + 6, 5k + 3, \dots, 3, 1, \dots, 5k + 1]$
$(0, 5, 1, 5k + 2)$	$[0, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 8, \dots, 3, 5, \dots, 5k + 5, 5k + 7]$
$(0, 6, 1, 5k + 2)$ P	$[0, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 8, \dots, 3, 5, \dots, 5k + 5, 5k + 7, 5k + 9]$

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(0, 7, 1, 5k + 2)$	$[0, 2, 4, \dots, 5k + 4, 5k + 6, 5k + 9, 5k + 7, \dots, 7, 5, \dots, 5k + 10, 5k + 8, \dots, 3, 1, \dots, 5k + 1]$
$(0, 4, 2, 5k + 2)$	P $[0, \dots, 5k + 5, 5k + 7, \dots, 2, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 3, 5k + 6, 5k + 8]$
$(0, 5, 2, 5k + 2)$	$[0, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 9, 5k + 7, 5k + 5, \dots, 5, 3, \dots, 5k + 8]$
$(0, 6, 2, 5k + 2)$	P $[0, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 9, 5k + 7, 5k + 5, \dots, 5, 3, \dots, 5k + 8, 5k + 10]$
$(0, 4, 3, 5k + 2)$	$[0, \dots, 5k + 5, 5k + 8, 5k + 6, 5k + 9, 5k + 7, \dots, 2, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 3]$
$(0, 5, 3, 5k + 2)$	$[0, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 9, 5k + 7, 5k + 10, 5k + 8, \dots, 3, 5, \dots, 5k + 5]$
$(0, 4, 1, 5k + 6)$	$[0, \dots, 5k + 10, 5k + 7, \dots, 2, 4, \dots, 5k + 9, 5k + 11, 5k + 6, 5k + 8, \dots, 3, 1, \dots, 5k + 1]$
$(0, 4, 2, 5k + 1)$	$[0, 2, \dots, 5k + 7, 5k + 5, \dots, 5, 3, \dots, 5k + 3, 5k + 6, 5k + 4, \dots, 4, 1, \dots, 5k + 1]$
$(0, 4, 3, 5k + 1)$	$[0, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 1, 5k + 3, \dots, 3, 5, \dots, 5k + 5, 5k + 8, 5k + 6]$
$(0, 5, 1, 5k + 1)$	$[0, 2, 4, \dots, 5k + 4, 5k + 7, \dots, 7, 5, \dots, 5k + 5, 5k + 3, \dots, 3, 1, \dots, 5k + 6]$
$(0, 5, 2, 5k + 1)$	$[0, 2, 5, \dots, 5k + 5, 5k + 7, \dots, 7, 4, \dots, 5k + 4, 5k + 6, 5k + 8, \dots, 3, 1, \dots, 5k + 1]$
$(0, 5, 3, 5k + 1)$	$[0, 2, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 8, 5k + 5, \dots, 5, 7, \dots, 5k + 7, 5k + 9, 5k + 6]$
$(0, 6, 2, 5k + 1)$	$[0, 2, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 8, 5k + 6, 5k + 9, 5k + 7, \dots, 7, 5, \dots, 5k + 5]$
$(0, 4, 1, 5k)$	P $[0, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 1, 5k + 3, \dots, 3, 5, \dots, 5k + 5]$
$(0, 4, 2, 5k)$	P $[0, 2, \dots, 5k + 2, 5k + 5, \dots, 5, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 4, \dots, 5k + 4, 5k + 6]$
$(0, 4, 3, 5k)$	$[0, 2, 5, \dots, 5k + 5, 5k + 7, \dots, 7, 4, \dots, 5k + 4, 5k + 6, 5k + 3, \dots, 3, 1, \dots, 5k + 1]$
$(0, 4, 4, 5k)$	P $[0, 2, 5, \dots, 5k + 5, 5k + 7, \dots, 7, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 3, 5k + 6, 5k + 8]$
$(0, 5, 1, 5k + 5)$	$[0, \dots, 5k + 10, 5k + 8, \dots, 3, 1, \dots, 5k + 11, 5k + 9, 5k + 7, 5k + 4, \dots, 4, 2, \dots, 5k + 2]$
$(0, 5, 2, 5k + 5)$	$[0, \dots, 5k + 5, 5k + 3, \dots, 3, 1, \dots, 5k + 11, 5k + 8, 5k + 10, 5k + 12, 5k + 9, \dots, 4, 2, \dots, 5k + 7]$
$(0, 5, 3, 5k)$	$[0, \dots, 5k, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 3, 5k + 6, 5k + 8, 5k + 5, 5k + 7]$
$(0, 6, 1, 5k + 5)$	$[0, 2, \dots, 5k + 12, 5k + 10, \dots, 5, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 4, \dots, 5k + 9, 5k + 11, 5k + 6, 5k + 8]$
$(0, 4, 1, 5k + 4)$	P $[0, 2, \dots, 5k + 7, 5k + 5, \dots, 5, 3, \dots, 5k + 8, 5k + 6, \dots, 1, 4, \dots, 5k + 9]$
$(0, 4, 2, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 7, 5k + 10, 5k + 8, \dots, 3, 1, \dots, 5k + 6, 5k + 9, \dots, 4, 2, \dots, 5k + 2]$
$(0, 4, 3, 5k + 4)$	P $[0, \dots, 5k + 10, 5k + 7, \dots, 2, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 8, 5k + 6, 5k + 9, 5k + 11]$
$(0, 5, 1, 5k + 4)$	$[0, 2, 4, \dots, 5k + 9, 5k + 6, \dots, 1, 3, \dots, 5k + 8, 5k + 10, \dots, 5, 7, \dots, 5k + 7]$
$(0, 6, 1, 5k + 4)$	$[0, \dots, 5k + 10, 5k + 8, 5k + 6, 5k + 11, 5k + 9, 5k + 7, \dots, 2, 4, \dots, 5k + 4, 5k + 1, \dots, 1, 3, \dots, 5k + 3]$
$(0, 5, 2, 5k + 4)$	$[0, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 11, 5k + 9, 5k + 7, 5k + 10, \dots, 5, 3, \dots, 5k + 8]$

## PROPOSITION 3.9

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(1, 0, 3t + 5, 3)$	$[0, 3, 6, 1, 4, 9, \dots, 3t + 9, 3t + 8, \dots, 2, 7, \dots, 3t + 7]$
$(1, 0, 3t + 6, 3)$	$[0, 3, 6, 1, 4, 9, \dots, 3t + 9, 3t + 10, \dots, 7, 2, \dots, 3t + 8]$
$(1, 0, 3t + 7, 3)$ P	$[0, 3, 6, 1, 4, 9, \dots, 3t + 9, 3t + 10, \dots, 7, 2, \dots, 3t + 11]$
$(1, 0, 3t + 5, 4)$ P	$[0, 3, 8, \dots, 3t + 8, 3t + 9, \dots, 6, 1, 4, 7, 2, 5, 10, \dots, 3t + 10]$
$(1, 0, 3t + 6, 4)$	$[0, 3, 8, \dots, 3t + 11, 3t + 10, \dots, 10, 5, 2, 7, 4, 1, 6, \dots, 3t + 9]$
$(1, 0, 3t + 7, 4)$	$[0, 3, 8, \dots, 3t + 11, 3t + 12, \dots, 6, 1, 4, 7, 2, 5, 10, \dots, 3t + 10]$
$(1, 0, 4, 5k + 1)$	$[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 5, \dots, 5, 2, \dots, 5k + 2]$
$(1, 0, 5, 5k + 1)$	$[0, 3, \dots, 5k + 3, 5k + 6, 5k + 5, \dots, 5, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 1]$
$(1, 0, 6, 5k + 1)$	$[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 7, 5k + 8, 5k + 5, \dots, 5, 2, \dots, 5k + 2]$
$(1, 0, 7, 5k + 1)$	$[0, 3, \dots, 5k + 3, 5k + 6, 5k + 9, 5k + 8, 5k + 5, \dots, 5, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 1]$
$(1, 0, 3, 5k + 2)$	$[0, \dots, 5k + 5, 5k + 2, \dots, 2, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 4]$
$(1, 0, 5, 5k + 2)$	$[0, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 5, \dots, 5k + 5, 5k + 8, \dots, 3, 6, \dots, 5k + 6]$
$(1, 0, 6, 5k + 2)$ P	$[0, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 5, \dots, 5k + 5, 5k + 8, \dots, 3, 6, \dots, 5k + 6, 5k + 9]$
$(1, 0, 7, 5k + 7)$	$[0, 3, 6, 1, 4, \dots, 5k + 14, 5k + 15, 5k + 12, \dots, 2, 5, \dots, 5k + 10, 5k + 13, \dots, 8, 11, \dots, 5k + 11]$
$(1, 0, 4, 5k + 5)$	$[0, \dots, 5k + 10, 5k + 7, \dots, 2, 1, 4, \dots, 5k + 9, 5k + 6, \dots, 6, 3, \dots, 5k + 8]$
$(1, 0, 5, 5k + 5)$	$[0, 3, \dots, 5k + 8, 5k + 11, \dots, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 5, \dots, 5k + 10, 5k + 9]$
$(1, 0, 6, 5k + 5)$	$[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 9, 5k + 12, \dots, 2, 5, \dots, 5k + 10, 5k + 11, 5k + 8]$
$(1, 0, 7, 5k + 5)$	$[0, 3, \dots, 5k + 8, 5k + 11, \dots, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 5, \dots, 5k + 10, 5k + 13, 5k + 12, 5k + 9]$
$(1, 0, 3, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 8, \dots, 3, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 6]$
$(1, 0, 4, 5k + 4)$ P	$[0, 3, \dots, 5k + 8, 5k + 5, \dots, 5, 2, \dots, 5k + 7, 5k + 6, \dots, 1, 4, \dots, 5k + 9]$
$(1, 0, 5, 5k + 4)$ P	$[0, 3, \dots, 5k + 8, 5k + 9, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 5, \dots, 5k + 10]$
$(1, 0, 6, 5k + 4)$	$[0, 3, \dots, 5k + 8, 5k + 5, \dots, 5, 2, \dots, 5k + 7, 5k + 10, 5k + 11, 5k + 6, 5k + 9, \dots, 4, 1, \dots, 5k + 1]$
$(1, 0, 7, 5k + 4)$	$[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 9, 5k + 12, \dots, 2, 5, \dots, 5k + 5, 5k + 8, 5k + 11, 5k + 10]$
$(1, 0, 5, 5k + 3)$	$[0, 3, \dots, 5k + 8, 5k + 9, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 5, \dots, 5k + 5]$
$(1, 0, 6, 5k + 3)$ P	$[0, 3, \dots, 5k + 3, 5k + 6, \dots, 1, 4, \dots, 5k + 9, 5k + 8, 5k + 5, \dots, 5, 2, \dots, 5k + 7, 5k + 10]$

## PROPOSITION 3.10

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(1, 1, 3t + 3, 1)$	$[0, \dots, 3t, 3t + 5, \dots, 2, 1, \dots, 3t + 4, 3t + 6, 3t + 3]$
$(1, 1, 3t + 4, 1)$	$[0, 3, 2, \dots, 3t + 5, 3t + 7, \dots, 1, 6, \dots, 3t + 6]$
$(1, 1, 3t + 5, 1)$	$[0, 3, 2, \dots, 3t + 8, 3t + 6, \dots, 6, 1, \dots, 3t + 7]$
$(1, 1, 3t + 5, 2)$	$[0, \dots, 3t + 9, 3t + 4, \dots, 1, 2, \dots, 3t + 2, 3t + 7, 3t + 5, 3t + 8]$
$(1, 1, 3t + 5, 3)$	$[0, 5, 2, 3, 8, \dots, 3t + 8, 3t + 10, \dots, 1, 6, \dots, 3t + 9]$
$(1, 1, 2, 5k)$	$[0, \dots, 5k, 5k + 3, \dots, 3, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 1, \dots, 5k + 1]$
$(1, 1, 2, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 2, \dots, 2, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 4, \dots, 5k + 4]$
$(1, 1, 2, 5k + 3)$	$[0, \dots, 5k + 5, 5k + 3, \dots, 3, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 6]$
$(1, 1, 3, 5k)$	$[0, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 4, \dots, 5k + 4, 5k + 5, \dots, 5, 2, \dots, 5k + 2]$
$(1, 1, 3, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 2, \dots, 2, 3, \dots, 5k + 3, 5k + 6, 5k + 4, \dots, 4, 1, \dots, 5k + 1]$
$(1, 1, 3, 5k + 2)$	$[0, \dots, 5k + 5, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 3, \dots, 3, 2, \dots, 5k + 2]$
$(1, 1, 3, 5k + 3)$	$[0, \dots, 5k + 5, 5k + 8, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 3, \dots, 5k + 3]$
$(1, 1, 3, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 7, \dots, 2, 1, 4, \dots, 5k + 9, 5k + 6, \dots, 6, 3, \dots, 5k + 8]$
$(1, 1, 4, 5k)$	$[0, 3, \dots, 5k + 3, 5k + 6, 5k + 4, \dots, 4, 1, \dots, 5k + 1, 5k + 2, \dots, 2, 5, \dots, 5k + 5]$

## PROPOSITION 3.11

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(1, 2, 3t + 2, 1)$	$[0, \dots, 3t + 3, 3t + 5, \dots, 2, 1, \dots, 3t + 1, 3t + 6, 3t + 4]$
$(1, 2, 3t + 3, 1)$	$[0, 2, \dots, 3t + 2, 3t + 7, \dots, 1, 3, \dots, 3t + 6, 3t + 5]$
$(1, 2, 3t + 4, 1)$	P $[0, 2, \dots, 3t + 2, 3t + 7, \dots, 1, 3, \dots, 3t + 6, 3t + 5, 3t + 8]$
$(1, 2, 3t + 3, 2)$	$[0, 2, 5, 3, 8, \dots, 3t + 8, 3t + 7, \dots, 1, 6, \dots, 3t + 6]$
$(1, 2, 3t + 4, 2)$	P $[0, 2, 5, 3, 8, \dots, 3t + 8, 3t + 7, \dots, 1, 6, \dots, 3t + 9]$
$(1, 2, 3t + 5, 2)$	P $[0, 2, 5, 3, 8, \dots, 3t + 8, 3t + 9, \dots, 6, 1, \dots, 3t + 10]$
$(1, 2, 1, 5k)$	P $[0, \dots, 5k, 5k + 2, \dots, 2, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 4, \dots, 5k + 4]$
$(1, 2, 1, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 3, \dots, 3, 1, \dots, 5k + 1]$
$(1, 2, 1, 5k + 2)$	P $[0, \dots, 5k + 5, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 3, \dots, 3, 1, \dots, 5k + 6]$
$(1, 2, 1, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 8, \dots, 3, 1, \dots, 5k + 6, 5k + 7, \dots, 2, 4, \dots, 5k + 4]$
$(1, 2, 2, 5k + 2)$	P $[0, \dots, 5k + 5, 5k + 6, 5k + 3, \dots, 3, 1, \dots, 5k + 1, 5k + 4, \dots, 4, 2, \dots, 5k + 7]$
$(1, 2, 2, 5k + 3)$	P $[0, \dots, 5k + 5, 5k + 3, \dots, 3, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 8]$
$(1, 2, 2, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 7, \dots, 2, 3, \dots, 5k + 8, 5k + 6, 5k + 9, \dots, 4, 1, \dots, 5k + 1]$
$(1, 2, 3, 5k + 3)$	P $[0, 2, 5, \dots, 5k + 5, 5k + 8, \dots, 3, 1, \dots, 5k + 6, 5k + 7, \dots, 7, 4, \dots, 5k + 9]$

## PROPOSITION 3.12

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(1, 2v + 6, 0, 1)$	$[0, 2, 4, 6, 1, \dots, 2v + 7, 2v + 8, \dots, 8]$
$(1, 2v + 7, 0, 1)$	$[0, 2, 4, 6, 1, \dots, 2v + 9, 2v + 8, \dots, 8]$
$(1, 3, 0, 5k)$	$[0, \dots, 5k, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 3, \dots, 3, 1, \dots, 5k + 1]$
$(1, 3, 0, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 3, \dots, 3, 1, \dots, 5k + 1, 5k + 2, \dots, 2, 4, \dots, 5k + 4]$
$(1, 3, 0, 5k + 3)$	$[0, \dots, 5k + 5, 5k + 7, \dots, 2, 4, \dots, 5k + 4, 5k + 3, \dots, 3, 1, \dots, 5k + 6]$
$(1, 4, 0, 5k)$	$[0, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 5, \dots, 5k + 5, 5k + 3, \dots, 3, 1, \dots, 5k + 1]$
$(1, 4, 0, 5k + 1)$ P	$[0, \dots, 5k + 5, 5k + 3, \dots, 3, 1, \dots, 5k + 1, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 6]$
$(1, 4, 0, 5k + 2)$	$[0, 1, \dots, 5k + 6, 5k + 4, \dots, 4, 2, \dots, 5k + 7, 5k + 5, \dots, 5, 3, \dots, 5k + 3]$
$(1, 5, 0, 5k)$	$[0, \dots, 5k, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 6, 5k + 5, 5k + 3, \dots, 3, 1, \dots, 5k + 1]$
$(1, 5, 0, 5k + 2)$	$[0, 2, \dots, 5k + 7, 5k + 5, \dots, 5, 4, \dots, 5k + 4, 5k + 6, 5k + 8, \dots, 3, 1, \dots, 5k + 1]$
$(1, 6, 0, 5k)$	$[0, \dots, 5k, 5k + 2, \dots, 2, 4, \dots, 5k + 4, 5k + 6, 5k + 7, 5k + 5, 5k + 3, \dots, 3, 1, \dots, 5k + 1]$
$(1, 6, 0, 5k + 2)$	$[0, 2, 1, 3, \dots, 5k + 8, 5k + 6, \dots, 6, 4, \dots, 5k + 9, 5k + 7, \dots, 7, 5, \dots, 5k + 5]$

## LEMMA 3.13

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(2, 0, 2, 5k)$ P	$[0, \dots, 5k, 5k + 3, \dots, 3, 2, \dots, 5k + 2, 5k + 1, \dots, 1, 4, \dots, 5k + 4]$
$(2, 0, 2, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 2, \dots, 2, 1, \dots, 5k + 1, 5k + 4, \dots, 4, 3, \dots, 5k + 3]$
$(2, 0, 2, 5k + 2)$ P	$[0, \dots, 5k + 5, 5k + 2, \dots, 2, 3, \dots, 5k + 3, 5k + 4, \dots, 4, 1, \dots, 5k + 6]$
$(2, 0, 2, 5k + 3)$ P	$[0, \dots, 5k + 5, 5k + 4, \dots, 4, 1, \dots, 5k + 6, 5k + 3, \dots, 3, 2, \dots, 5k + 7]$
$(2, 0, 2, 5k + 4)$ P	$[0, \dots, 5k + 5, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 3, \dots, 5k + 8]$
$(2, 1, 1, 5k)$	$[0, \dots, 5k, 5k + 1, \dots, 1, 4, \dots, 5k + 4, 5k + 2, \dots, 2, 3, \dots, 5k + 3]$
$(2, 1, 1, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 2, \dots, 2, 1, \dots, 5k + 1, 5k + 3, \dots, 3, 4, \dots, 5k + 4]$
$(2, 1, 1, 5k + 2)$	$[0, \dots, 5k + 5, 5k + 2, \dots, 2, 1, \dots, 5k + 6, 5k + 4, \dots, 4, 3, \dots, 5k + 3]$
$(2, 1, 1, 5k + 3)$	$[0, \dots, 5k + 5, 5k + 7, \dots, 2, 1, \dots, 5k + 6, 5k + 3, \dots, 3, 4, \dots, 5k + 4]$
$(2, 1, 1, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 8, \dots, 3, 4, \dots, 5k + 4, 5k + 6, \dots, 1, 2, \dots, 5k + 7]$
$(2, 2, 0, 5k)$ P	$[0, \dots, 5k, 5k + 1, \dots, 1, 3, \dots, 5k + 3, 5k + 2, \dots, 2, 4, \dots, 5k + 4]$
$(2, 2, 0, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 3, \dots, 3, 4, \dots, 5k + 4, 5k + 2, \dots, 2, 1, \dots, 5k + 1]$
$(2, 2, 0, 5k + 2)$ P	$[0, \dots, 5k + 5, 5k + 3, \dots, 3, 4, \dots, 5k + 4, 5k + 2, \dots, 2, 1, \dots, 5k + 6]$
$(2, 2, 0, 5k + 3)$ P	$[0, \dots, 5k + 5, 5k + 6, \dots, 1, 3, \dots, 5k + 3, 5k + 4, \dots, 4, 2, \dots, 5k + 7]$
$(2, 2, 0, 5k + 4)$ P	$[0, \dots, 5k + 5, 5k + 4, \dots, 4, 2, \dots, 5k + 7, 5k + 6, \dots, 1, 3, \dots, 5k + 8]$

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(2, 0, 3t + 3, 1)$	$[0, 3, 2, \dots, 3t + 5, 3t + 6, \dots, 6, 1, \dots, 3t + 4]$
$(2, 0, 3t + 4, 1)$	P $[0, 3, 2, \dots, 3t + 5, 3t + 6, \dots, 6, 1, \dots, 3t + 7]$
$(2, 0, 3t + 5, 1)$	$[0, 3, 2, \dots, 3t + 8, 3t + 7, \dots, 1, 6, \dots, 3t + 6]$
$(2, 0, 3, 5k)$	P $[0, 3, \dots, 5k + 3, 5k + 4, \dots, 4, 1, \dots, 5k + 1, 5k + 2, \dots, 2, 5, \dots, 5k + 5]$
$(2, 0, 3, 5k + 1)$	P $[0, 3, \dots, 5k + 3, 5k + 2, \dots, 2, 5, \dots, 5k + 5, 5k + 4, \dots, 4, 1, \dots, 5k + 6]$
$(2, 0, 3, 5k + 3)$	$[0, 3, \dots, 5k + 8, 5k + 7, \dots, 2, 5, \dots, 5k + 5, 5k + 6, \dots, 1, 4, \dots, 5k + 4]$
$(2, 1, 2, 5k + 2)$	$[0, 3, \dots, 5k + 3, 5k + 5, \dots, 5, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 1, \dots, 5k + 6]$
$(2, 1, 2, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 7, \dots, 2, 3, \dots, 5k + 8, 5k + 9, 5k + 6, \dots, 1, 4, \dots, 5k + 4]$
$(2, 2, 1, 5k + 3)$	P $[0, \dots, 5k + 5, 5k + 3, \dots, 3, 4, \dots, 5k + 4, 5k + 7, \dots, 2, 1, \dots, 5k + 6, 5k + 8]$
$(2, 3, 0, 5k + 2)$	$[0, 2, \dots, 5k + 7, 5k + 5, \dots, 5, 4, \dots, 5k + 4, 5k + 3, \dots, 3, 1, \dots, 5k + 6]$
$(2, 3, 0, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 7, 5k + 9, \dots, 4, 3, \dots, 5k + 8, 5k + 6, \dots, 1, 2, \dots, 5k + 2]$

## LEMMA 3.14

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(3, 0, 1, 5k)$	$[0, \dots, 5k, 5k + 1, \dots, 1, 4, \dots, 5k + 4, 5k + 3, \dots, 3, 2, \dots, 5k + 2]$
$(3, 0, 1, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 4, \dots, 4, 1, \dots, 5k + 1, 5k + 2, \dots, 2, 3, \dots, 5k + 3]$
$(3, 0, 1, 5k + 2)$	$[0, \dots, 5k + 5, 5k + 6, \dots, 1, 4, \dots, 5k + 4, 5k + 3, \dots, 3, 2, \dots, 5k + 2]$
$(3, 0, 1, 5k + 3)$	$[0, \dots, 5k + 5, 5k + 6, \dots, 1, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 3, \dots, 5k + 3]$
$(3, 0, 1, 5k + 4)$	P $[0, \dots, 5k + 5, 5k + 6, \dots, 1, 2, \dots, 5k + 7, 5k + 4, \dots, 4, 3, \dots, 5k + 8]$
$(3, 1, 0, 5k)$	$[0, \dots, 5k, 5k + 1, \dots, 1, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 3, \dots, 5k + 3]$
$(3, 1, 0, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 4, \dots, 4, 3, \dots, 5k + 3, 5k + 1, \dots, 1, 2, \dots, 5k + 2]$
$(3, 1, 0, 5k + 2)$	$[0, \dots, 5k + 5, 5k + 6, \dots, 1, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 3, \dots, 5k + 3]$
$(3, 1, 0, 5k + 4)$	P $[0, \dots, 5k + 5, 5k + 4, \dots, 4, 3, \dots, 5k + 8, 5k + 6, \dots, 1, 2, \dots, 5k + 7]$
$(4, 0, 0, 5k)$	P $[0, \dots, 5k, 5k + 1, \dots, 1, 2, \dots, 5k + 2, 5k + 3, \dots, 3, 4, \dots, 5k + 4]$
$(4, 0, 0, 5k + 1)$	$[0, \dots, 5k + 5, 5k + 4, \dots, 4, 3, \dots, 5k + 3, 5k + 2, \dots, 2, 1, \dots, 5k + 1]$
$(4, 0, 0, 5k + 2)$	$[0, \dots, 5k + 5, 5k + 6, \dots, 1, 2, \dots, 5k + 2, 5k + 3, \dots, 3, 4, \dots, 5k + 4]$
$(4, 0, 0, 5k + 4)$	$[0, \dots, 5k + 5, 5k + 4, \dots, 4, 3, \dots, 5k + 8, 5k + 7, \dots, 2, 1, \dots, 5k + 6]$
$(4, 1, 0, 5k + 3)$	$[0, \dots, 5k + 5, 5k + 6, \dots, 1, 2, \dots, 5k + 2, 5k + 4, \dots, 4, 3, \dots, 5k + 8, 5k + 7]$
$(5, 0, 0, 5k + 3)$	$[0, \dots, 5k + 5, 5k + 4, \dots, 4, 3, \dots, 5k + 8, 5k + 7, 5k + 6, \dots, 1, 2, \dots, 5k + 2]$

## THEOREM 2.4

$(a, b, c, d)$	$r\{1^a, 2^b, 3^c, 5^d\}$
$(1, 3, 1, 5k + 3)$	$[0, \dots, 5k + 5, 5k + 8, 5k + 6, \dots, 1, 3, \dots, 5k + 3, 5k + 4, \dots, 4, 2, \dots, 5k + 7]$

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DICATAM - SEZ. MATEMATICA, UNIVERSITÀ DEGLI STUDI DI BRESCIA, VIA VALOTTI 9, I-25133 BRESCIA, ITALY  
*E-mail address:* [anita.pasotti@ing.unibs.it](mailto:anita.pasotti@ing.unibs.it)

DEPARTAMENTO DE MATEMÁTICA, UNIVERSIDADE DE BRASÍLIA - ICC CENTRO, 70910-900 BRASÍLIA - DF, BRAZIL  
*E-mail address:* [pellegrini@unb.br](mailto:pellegrini@unb.br)